CONVEX GEOMETRY OF COMPLETELY POSITIVE MAPS

IGOR KLEP

A linear map Φ between matrix spaces is positive if it maps positive semidefinite matrices to positive semidefinite ones, and is called completely positive if all its ampliations $I_n \otimes \Phi$ are positive. To each (completely) positive map one can associate a pair of nested (free) spectrahedra, and vice-versa. Spectrahedra are solution sets to linear matrix inequalities (LMIs). LMIs are ubiquitous in real algebraic geometry, semidefinite programming, control theory and signal processing. LMIs with matrix unknowns on the other hand are central to the theories of completely positive maps and operator algebras, operator systems and spaces. The talk will employ convex geometry and dilation theory to investigate the question of how far a positive map is from being completely positive. Along the way we will observe that any tuple of $d \times d$ matrices dilates to a tuple of commuting matrices, where the norm increase is bounded in terms of d alone.