THE INVERSE EIGENVALUE PROBLEM OF A GRAPH AND ZERO FORCING

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Inverse eigenvalue problems appear in various contexts throughout mathematics and engineering, and refer to determining whether not there is a matrix with a prescribed structure (e.g., tridiagonal) and prescribed spectral property (e.g., having an eigenvalue of given multiplicity, or having few distinct eigenvalues). For a given simple graph G, the matrices in $\mathcal{S}(G)$ are real, symmetric, and have off-diagonal nonzero entries exactly where G has edges. The inverse eigenvalue problem of G is to determine the collection of all possible spectra (multisets of eigenvalues) for matrices in $\mathcal{S}(G)$. The background of this problem will be described, together with techniques such as the fundamental work of Colin de Verdière using the Strong Arnold Property and recent extensions of the Strong Arnold Property that target a better understanding of all possible spectra and their associated multiplicities.

The zero forcing number $Z(G)$ was introduced independently in several areas, including as an upper bound for the maximum eigenvalue multiplicity of matrices in $\mathcal{S}(G)$. Zero forcing refers to forcing zeros in an eigenvector and is described combinatorially as a coloring game on a graph. Each vertex is colored blue or white and a blue vertex v can color a white vertex w blue if w is the only white neighbor of v. Zero forcing, variants, and their applications will be described.

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