

Entrywise positivity preservers: covariance estimation, symmetric function identities, and a novel graph invariant

Apoorva Khare (Indian Institute of Science, and Analysis & Probability Research Group, Bangalore).

Which functions preserve positive semidefiniteness (psd) when applied entrywise to psd matrices? This question has a long history beginning with Schur, Schoenberg, and Rudin, and has also recently received renewed attention due to applications in high-dimensional statistics. After explaining some of these developments, I will present a selection of results on entrywise positivity preservers. We begin with early work of Schoenberg (and Bochner, Weil, ...) which led from metric (spherical) geometry to matrix positivity and its preservers. Next come some of the contributions of Loewner, Karlin, and their students: FitzGerald, Horn, Micchelli, and Pinkus, on entrywise maps in all dimensions and in a fixed dimension. I will end with some recent results that describe novel connections to symmetric function theory and combinatorics.

(Partly based on joint works with Alexander Belton, Dominique Guillot, Mihai Putinar, Bala Rajaratnam, and Terence Tao.)